Symmetry and Asymmetry from Local Phase

Peter Kovesi

Department of Computer Science
The University of Western Australia
Nedlands W.A. 6907
pk@cs.uwa.edu.au

Abstract. Symmetry is an important mechanism by which we identify the structure of objects. Man-made objects, plants and animals are usually highly recognizable from the symmetry, or partial symmetries that they often exhibit. Two difficulties found in most symmetry detection algorithms are firstly, that they usually require objects to be segmented *prior* to any symmetry analysis, and secondly, that they do not provide any absolute measure of the degree of symmetry at any point in an image. This paper presents a new measure of symmetry that is based on the analysis of local frequency information. It is shown that points of symmetry and asymmetry give rise to easily recognized patterns of local phase. This phase information can be used to construct a contrast invariant measure of symmetry that does not require any prior recognition or segmentation of objects.

Keywords: Computer Vision, Symmetry, Asymmetry, Local Phase, Quadrature Filters

1 Introduction

Under the most general definition of symmetry an object is considered symmetric if it remains invariant under some transformation. Two forms of symmetry that we can readily identify in images are bilateral symmetry and rotational symmetry. An object exhibits bilateral symmetry if it remains invariant with respect to reflection about some axis. An object has rotational symmetry if it remains invariant with respect to rotations about some axis. This paper will mainly consider bilateral symmetry and in the following discussion where the word symmetry is used it should be taken to mean bilateral symmetry.

Symmetry is an important mechanism by which we identify the structure of objects. Man-made objects, plants and animals are usually highly recognizable from the symmetry, or partial symmetries that they often exhibit. A limited number of approaches have been tried in the detection of symmetry in images. A fundamental weakness found in most is that they require objects to be segmented *prior* to any symmetry analysis. For example Atallah [1] describes an algorithm that requires objects to be represented in terms of points, line segments and circles. Morphological techniques such as medial axis transforms, thinning, and 'grass fire' algorithms can only be applied to binary objects. A survey of these approaches is provided by Xie [12]. A difficulty with morphological approaches is that they are very sensitive to small variations in the outlines of objects; a notch in an object contour will propagate several symmetry axes, complicating the representation of the object. Brady and Asada [2] attempt to overcome these problems by using smoothed object contours as input to an algorithm that is effectively morphological in nature.

Reisfeld et al. [10] provide one of the few approaches to symmetry that does not require object recognition or segmentation. Opposing pairs of points within some distance of a location in the image are considered with respect to the direction and strength of the intensity gradients at these points. At each location in the image a weighted sum of the degree of symmetry of the surrounding opposing pairs of points is computed to obtain an overall symmetry measure. Each pair of points contributes to the measure of symmetry according to the symmetry of the directions and magnitudes of their intensity gradients, and to the strength of the intensity gradients themselves. An objection to Reisfeld et al.'s measure of symmetry is that it depends on the contrast of the feature in addition to its geometric shape. A bright circle will be considered to be more 'symmetric' than a low contrast one. Thus, we have no absolute sense of the degree of symmetry of an object, all one obtains are locations in the image where symmetry is locally maximal.

2 A Frequency Approach to Symmetry

An important aspect of symmetry is the periodicity that it implies in the structure of the object that one is looking at. Accordingly it is perhaps natural that one should use a frequency based approach in attempting to recognize and analyze symmetry in images. Indeed, an inspection of the Fourier series of some simple functions makes this very apparent. At points of symmetry and asymmetry we find readily identifiable patterns of phase. Figure 1 shows the Fourier series representation of both a square wave and a triangular wave. We can see that the axis of symmetry corresponds to the point where all the frequency components are at either the minimum or maximum points in their cycles, that is, where all the frequency components are at the most symmetric points in their cycles (the mid-point of the square wave and the peaks/troughs of the triangular wave). Similarly one can see that the axis of asymmetry corresponds to the point where all the frequency components are at the most asymmetric points in their cycles; the inflection point (the steps on the square wave and the mid-point of the ramp on the triangular wave).

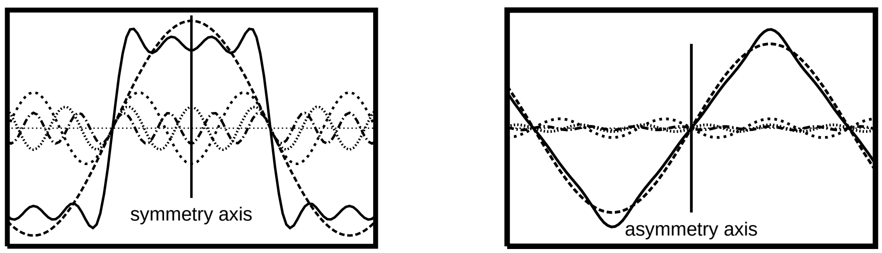


Fig. 1. Phase patterns at points of symmetry and asymmetry

It should be noted that here we are only considering symmetry and asymmetry of *intensity values* in images, that is, a low-level view of symmetry and asymmetry. Overall geometric symmetries that might exist in the image are not considered; this level of analysis requires the recognition of higher level structures in images which will not be considered here.

3 Localized Frequency Representation of a Signal

In this work the Wavelet Transform is used to obtain local frequency information. The use of the Wavelet Transform for frequency analysis was developed by Morlet et al. [8]. The basic idea behind wavelet analysis is that one uses a bank of filters to analyze the signal. The filters are all created from rescalings of the one wave shape, each scaling designed to pick out a particular band of frequencies from the signal being analyzed. An important point is that the scales of the filters vary geometrically, giving rise to a logarithmic frequency scale.

We are interested in calculating local frequency and, in particular, phase information in signals. To preserve phase information linear-phase filters must be used, that is, we must use wavelets that are in symmetric/anti-symmetric pairs. This constraint means that the work on orthogonal wavelets (which dominates much of the literature) is not applicable to us. Chui provides a proof that, with the exception of the Haar wavelet, one cannot have a wavelet of compact support that is both symmetric and orthogonal [3]. For our work we will follow the approach of Morlet, that is, using wavelets based on complex valued Gabor functions - sine and cosine waves, each modulated by a Gaussian. Using two filters in quadrature enables one to calculate the amplitude and phase of the signal for a particular scale/frequency at a given spatial location. However, rather than using Gabor filters we prefer to use *Log Gabor* functions as suggested by Field [4]; these are filters having a Gaussian transfer function when viewed on the logarithmic frequency scale. Log Gabor filters allow arbitrarily large bandwidth filters to be constructed while still maintaining a zero DC component in the even-symmetric filter. A zero DC value cannot be maintained in Gabor functions for bandwidths over 1 octave.

Analysis of a signal is done by convolving the signal with each of the quadrature pairs of wavelets. If we let I denote the signal and M_n^e and M_n^o denote the even-symmetric (cosine) and odd-symmetric (sine) wavelets at a scale n we can think of the responses of each quadrature pair of filters as forming a response vector,

$$[\ e_n(x), o_n(x)\ ] = [\ I(x) * M_n^e, I(x) * M_n^o\ ]\ .$$

The values $e_n(x)$ and $o_n(x)$ can be thought of as real and imaginary parts of complex valued frequency component. The amplitude of the transform at a given wavelet scale is given by

$$A_n(x) = \sqrt{e_n(x)^2 + o_n(x)^2}$$

and the phase is given by

$$\Phi_n(x) = atan2(e_n(x), o_n(x)).$$

At each point x in a signal we will have an array of these response vectors, one vector for each scale of filter. These response vectors form the basis of our localized representation of the signal as shown in Figure 2.

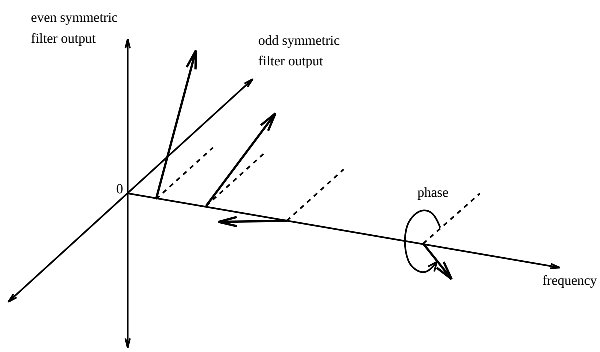


Fig. 2. An array of filter response vectors at a point in a signal can be represented as a series of vectors radiating out from the frequency axis. The amplitude specifies the length of each vector and the phase specifies its angle. Note that wavelet filters are scaled geometrically, hence their centre frequencies vary accordingly.

4 Symmetry and Phase

As shown in Figure 3 at a point of symmetry the absolute value of the even-symmetric filter outputs will be large and the absolute value of the odd-symmetric filter outputs will be small. Thus, a natural way to quantify symmetry is to take the absolute value of the even-symmetric filter output and subtract from it the absolute value of the odd-symmetric filter output. This corresponds to taking the absolute value of the cosine of the phase angle and subtracting from it the absolute value of the sine of the phase angle. This produces a function that varies between ± 1 and varies almost linearly with phase deviation, as shown in Figure 4

To combine information from filter responses over multiple scales a weighted average is formed. The difference of the absolute values of the even and odd filter responses at each scale is weighted by the magnitude of the filter response vector at each scale A_n. The sum of these weighted differences is then normalized by the sum of the magnitude of the filter response vectors over all scales. This produces the following equation:

$$\begin{aligned} Sym(x) &= \frac{\sum_n \lfloor A_n(x)[|\cos(\phi_n(x))| - |\sin(\phi_n(x))|] - T \rfloor}{\sum_n A_n(x) + \varepsilon} \\ &= \frac{\sum_n \lfloor [|e_n(x)| - |o_n(x)|] - T \rfloor}{\sum_n A_n(x) + \varepsilon}\ , \end{aligned}$$

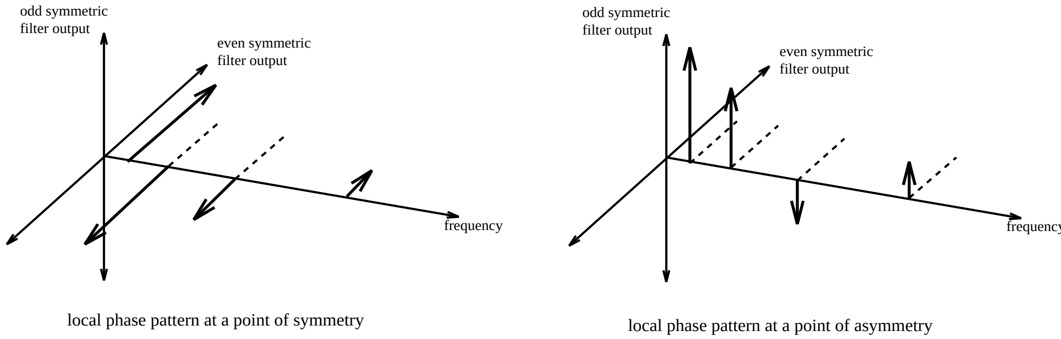


Fig. 3. At a point of symmetry the local phase pattern will be such that only even-symmetric filters will be responding, and at a point of asymmetry only odd-symmetric filters will be responding.

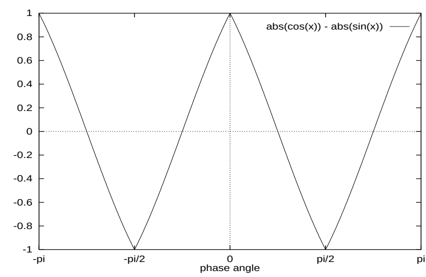


Fig. 4. Plot of the symmetry measure $|\cos(x)| - |\sin(x)|$.

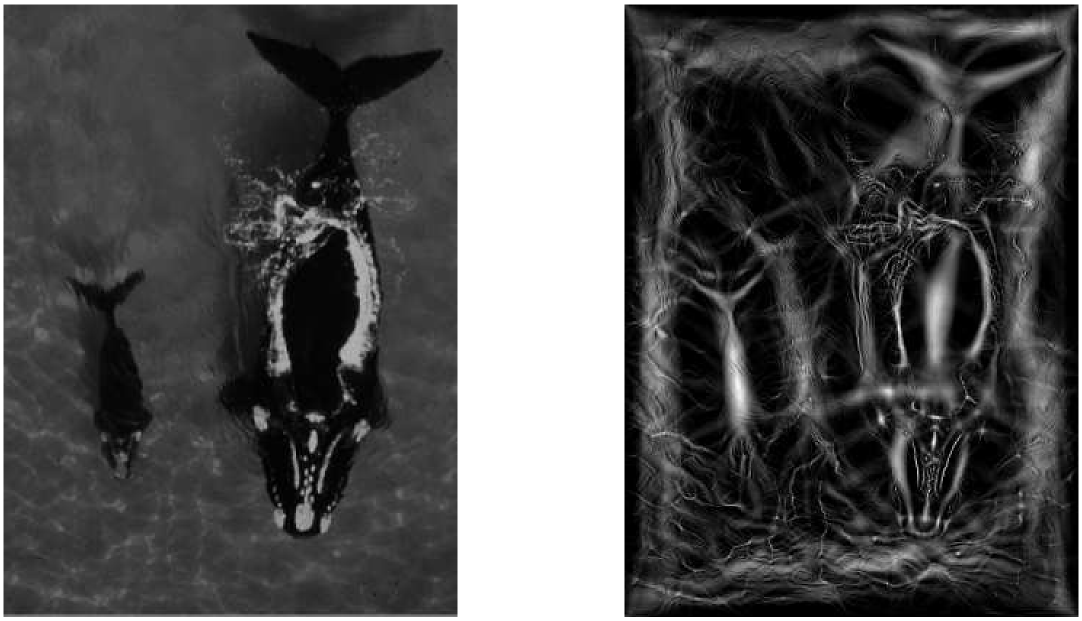
The term ε is a small constant to prevent division by zero in the case where the signal is uniform and no filter response is obtained. The factor T is a noise compensation term representing the maximum response that could be generated from noise alone in the signal. This factor is obtained by combining the estimated influence of noise on each of the filters. If one assumes the noise spectrum is flat the maximal effect of the noise on each of the filter outputs can be estimated from the mean amplitude response of the smallest scale filter as follows: Features in signals tend to be sparse but noise is everywhere. The regions where the smallest scale filter will be responding to features will be limited as its spatial extent is small, mostly the filter will be only responding to noise. Thus, the mean amplitude response will form an estimate of the mean influence of noise on the smallest scale filter. The noise response of other filters can then be estimated according to their bandwidths relative to that of the smallest scale filter. More details of this technique are described elsewhere by Kovesi [5, 6].

At points of high asymmetry the absolute values of the even and odd-symmetric filter outputs will be reversed from the symmetric case. The magnitude of the odd-symmetric filter output will be large and the even-symmetric filter output will be small. Thus, a measure of asymmetry can be expressed as

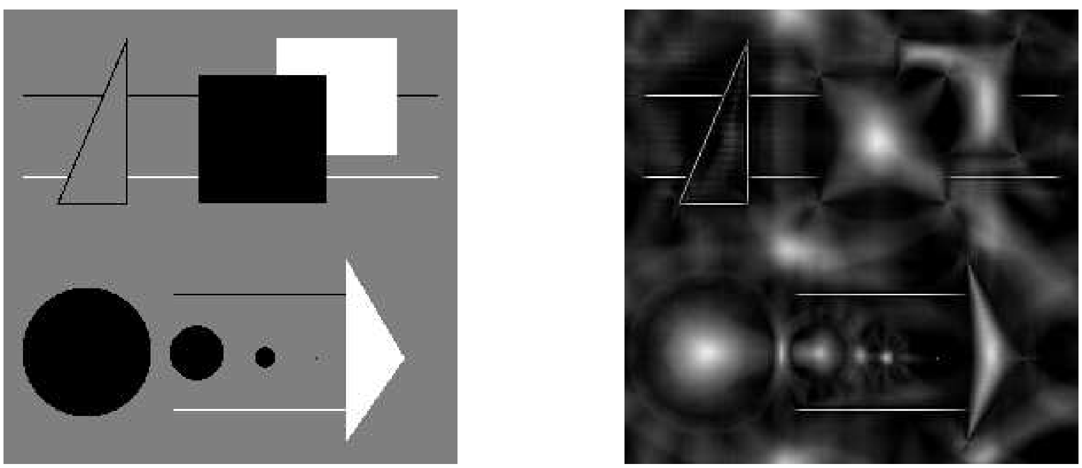
$$ASym(x) = \frac{\sum_n \lfloor [|o_n(x)| - |e_n(x)|] - T \rfloor}{\sum_n A_n(x) + \varepsilon}$$

These measures of symmetry and asymmetry can be related to the phase congruency model of feature perception [9, 11, 5, 6]. Symmetry, in some sense, represents a generalization of delta features and asymmetry represents a generalization of step edges. A delta feature starts off having all frequency components aligned in phase and in symmetry. As a delta feature is broadened into a rectangle function the alignment in phase of the higher frequency components starts to break down but the symmetry remains. Similarly, when a step function is gradually degraded to a trapezoidal function the phase congruency at the centre of the ramp progressively breaks down but all the frequency components remain asymmetric at that point.

The equations above only deal with symmetry/asymmetry in one dimension. One can extend this 1D analysis to 2D by applying the 1D analysis in multiple orientations and forming a weighted sum of the result. It should be noted that this approach only provides a basic extension to 2D in that no attention is paid to the type of symmetry that may be occurring at any point in an image. For example, no distinction is made between bilateral and radial symmetries, the symmetry measures are simply pooled over all orientations. Two examples of phase symmetry images obtained using this simple approach are shown in Figure 5. Ideally, some consideration to the way in which symmetry varies with orientation at each point in the image should be made. This would allow classification of bilateral and radial symmetries.



Whale image and corresponding phase symmetry image



Test image and corresponding phase symmetry image

Fig. 5. Two examples of phase symmetry images.

The results obtained from the phase symmetry measure can sometimes be counter-intuitive. This is because it measures local symmetry *to the exclusion of everything else.* The measure is invariant to the magnitude of the local contrast and so features that we might consider to be of little significance can be marked as having strong symmetry (see the wave patterns in the Whale image of Figure 5). Secondly, being a low-level measure that only considers local intensity values there is no distinction between foreground objects and background; it will faithfully report symmetries that occur in the spaces between foreground objects. This is in contrast to what we generally do when studying a scene, that is, we only consider the properties of the foreground objects which we have unconsciously segmented out from the scene. Thus, the output of the phase symmetry measure may not quite be what one 'wants' when one searches for 'symmetry' in an image. Perhaps this indicates we need to think more about what we *really* want when we say 'symmetry'; we may be talking about some other (undefined) quantity that incorporates other properties such as local contrast and foreground/background attributes. MATLAB code for reproducing the results presented here is provided by Kovesi [7].

5 Summary

It has been shown that local symmetry and asymmetry in image intensity patterns can be identified as being particular arrangements of phase. The new phase-based measures of symmetry and asymmetry that are presented here are significant in that they are low-level operators that do not require any prior object recognition or segmentation. They are also unique in that, unlike symmetry measures developed by others, they are dimensionless measures that provide an *absolute* sense of the degree of local symmetry or asymmetry independent of the image illumination or contrast.

References

1. J. R. Atallah. On symmetry detection. *IEEE Transactions on Computers*, C-34:663–666, 1985.
2. M. Brady and H. Asada. Smoothed local symmetries and their implementation. *The International Journal of Robotics Research*, 3(3):36–61, 1984.
3. C. K. Chui. *An Introduction to Wavelets*. Academic Press, San Diego, CA, 1992.
4. D. J. Field. Relations between the statistics of natural images and the response properties of cortical cells. *Journal of The Optical Society of America A*, 4(12):2379–2394, December 1987.
5. P. D. Kovesi. Image features from phase congruency. Technical Report 95/4, Department of Computer Science, The University of Western Australia, 1995. ftp: cs.uwa.edu.au /pub/techreports/95/4.ps.gz.
6. P. D. Kovesi. *Invariant Measures of Image Features From Phase Information*. PhD thesis, The University of Western Australia, May 1996.
7. P. D. Kovesi. MATLAB code for calculating phase congruency and phase symmetry/asymmetry, April 1996. http://www.cs.uwa.edu.au/∼pk/Research/research.html.
8. J. Morlet, G. Arens, E. Fourgeau, and D. Giard. Wave propagation and sampling theory - Part II: Sampling theory and complex waves. *Geophysics*, 47(2):222–236, February 1982.
9. M. C. Morrone and R. A. Owens. Feature detection from local energy. *Pattern Recognition Letters*, 6:303–313, 1987.
10. D. Reisfeld, H. Wolfson, and Y. Yeshurun. Detection of interest points using symmetry. In *Proceedings of the 3rd International Conference on Computer Vision*, pages 62–65, December 1990. Osaka, Japan.
11. S. Venkatesh and R. Owens. On the classification of image features. *Pattern Recognition Letters*, 11:339–349, 1990.
12. Y. Xia. Skeletonization via the realization of the fire front's propagation and extinction in digital binary shapes. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 11(10):1076–1086, 1989.

This article was processed using the LaTeX macro package with LLNCS style